

Contents

Preface	ix
0 Introduction	1
Exercises	11
1 Geometric Vectors	13
1.1 Addition of geometric vectors	18
1.2 Multiplication by a scalar	20
1.3 Subtraction of vectors	24
1.4 List of useful properties	25
1.5 The geometry of parallelograms	26
Exercises	30
2 Position Vectors and Components	33
2.1 Magnitude, unit vectors and hat notation	34
2.2 Parallel vectors	36
2.3 Position vectors and components	37
2.4 Length of a vector	43
2.5 Linear independence for two vectors	44
Exercises	49
3 Dot Products and Projections	53
3.1 Geometric definition of dot product	55
3.2 Algebraic definition of dot product	57
3.3 Angle between two vectors	61
3.4 Projections and orthogonal components	62
3.5 Another application to geometry in the plane	67
Exercises	69
4 Cross Products	71
4.1 Definition of cross product	72

4.2	List of useful properties	74
4.3	Method of expanding brackets	76
4.4	Geometric interpretation	76
4.5	Continuity and the right-hand orientation	81
	Exercises	83
5	Lines in Space	87
5.1	Parametric vector and scalar equations of a line	89
5.2	Cartesian equations of a line	91
5.3	Finding a line using two points	93
5.4	Distance from a point to a line	95
	Exercises	99
6	Planes in Space	103
6.1	Vector equation of a plane	105
6.2	Cartesian equation of a plane	107
6.3	Finding a plane using three points	109
6.4	Distance from a point to a plane	110
	Exercises	113
7	Systems of Linear Equations	115
7.1	Consistent and inconsistent systems	118
7.2	Parametric solutions	122
7.3	Augmented matrix of a system	124
7.4	Gaussian elimination	126
7.5	Reduced row echelon form	134
	Exercises	137
8	Matrix Operations	139
8.1	Addition, subtraction and scalar multiplication	142
8.2	Matrix multiplication	143
8.3	Connections with systems of equations	147
	Exercises	150
9	Matrix Inverses	153
9.1	Identity matrices and inverses	156
9.2	Inverses of two-by-two matrices	158
9.3	Powers of a matrix	160
9.4	Using row reduction to find the inverse	161
9.5	Using inverses to solve systems of equations	163
9.6	Elementary matrices	164

Exercises	169
10 Determinants	171
10.1 Determinant of a three-by-three matrix	173
10.2 Cross products revisited	177
10.3 Properties of determinants	178
10.4 Orientation of a triangle	181
Exercises	186
11 Eigenvalues and Eigenvectors	189
11.1 Existence of eigenvalues	196
11.2 Finding eigenvalues	203
11.3 Reflections and rotations in the plane	208
Exercises	216
12 Diagonalising a Matrix	219
12.1 An example which cannot be diagonalised	225
12.2 An example of a Markov process	228
12.3 The Jordan form of a matrix	235
Exercises	240
Index	243