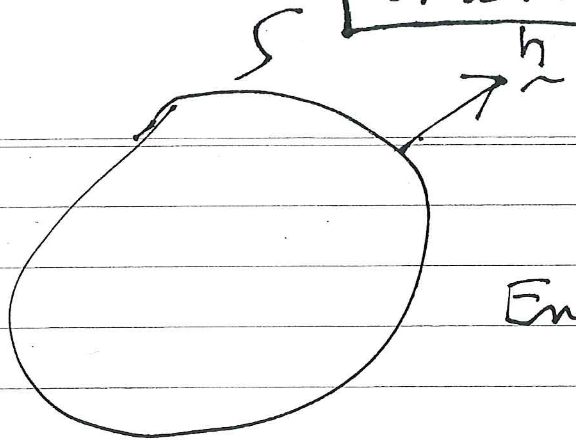
CONSERVATION OF MASS



CLOSED SURFACE S

Enclosing a volume V

$\rho = \rho(\underset{\sim}{x}, t)$ DENSITY

TOTAL MASS INSIDE V $= \iiint_V \rho \, dV$

NET RATE OF MASS FLOWING OUTWARS ACROSS S $= \iint_S \rho \underset{\sim}{u} \cdot \underset{\sim}{n} \, dS$

$\underset{\sim}{n}$ = UNIT NORMAL

$\underset{\sim}{u}$ = Velocity vector.

If NO SOURCES OF FLUID inside V

RATE OF CHANGE OF MASS =

— RATE OF FLOW

$$\Rightarrow \frac{d}{dt}\iiint_V \rho \, dV = -\iint_S \rho \underset{\sim}{u} \cdot \underset{\sim}{n} \, dS$$

[BY DIVERGENCE THM]

$$= -\iiint_V \nabla \cdot (\rho \underset{\sim}{u}) \, dV$$

$$\Rightarrow \iiint_V \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{u}) \right] dV = 0$$

Since this is valid for all volumes V

$\Rightarrow$ Integrand must be zero.

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{u}) = 0}$$