“GROUPOIDS, GRAPHS, AND ALGEBRAS
TITLES AND ABSTRACTS

Mini Courses

Aidan Sims (University of Wollongong)

Introduction to groupoids

This lecture will introduce the algebraic concept of a groupoid and also the concept of a Hausdorff topological groupoids, with particular emphasis on étale groupoids. We will explore a number of key constructions and examples.

Uniqueness theorems

This lecture will discuss the key uniqueness theorems for both Steinberg algebras and groupoid C^*-algebras, and describe some applications.

Reconstruction theory

This lecture will discuss how to reconstruct étale groupoids from C^*-algebraic data, and how to reconstruct an ample groupoid from algebraic data. We will also discuss some key applications.

Equivalence of groupoids
This lecture will describe the notion of equivalence of groupoids and its relationship to stable isomorphism and Kakutani equivalence for ample groupoids, and and relate this to Morita equivalence and to stable isomorphism of algebras and C^*-algebras.

Lisa Orloff Clark (Victoria University Wellington)

Steinberg algebras

This lecture will introduce the notion of an ample groupoid a special type of étale groupoid and describe the Steinberg algebra of an ample groupoid over a field K, including some key examples and structural features.

C^*-algebras of étale groupoids

This lecture will describe both the full and the reduced C^*-algebra of an étale groupoid, indicate how they relate to the complex Steinberg algebra, and outline some key examples and structural features.

Ideals and simplicity

This lecture will describe the notion of equivalence of groupoids and its relationship to stable isomorphism and Kakutani equivalence for ample groupoids, and and relate this to Morita equivalence and to stable isomorphism of algebras and C^*-algebras.

Pure infiniteness

This lecture will discuss locally contracting groupoids and what is known about when a Steinberg algebra or a groupoid C^*-algebra is purely infinite.

Plenary speakers

Karen Strung (Radboud Universiteit)

Dynamical systems, groupoids and the classification of C^*-algebras – Smale spaces

My lectures will discuss C^*-algebras from dynamical systems via groupoids and their relation to the classification program for C^*-algebras. Now that the classification by Elliott invariants of infinite dimensional simple separable unital C^*-algebras with finite nuclear dimension and which satisfy the UCT is complete, it becomes an interesting question to determine what classes of "naturally occurring" C^*-algebra are covered by the classification, as well as the range of the invariant for such classes.
In my first lecture, I will discuss groupoids associated to certain hyperbolic dynamical systems called Smale spaces. The talk will focus on joint work with Robin Deeley which shows that the C*-algebras associated to the so-called homoclinic groupoids of irreducible Smale spaces can be classified by the Elliott invariant.

Dynamical systems, groupoids and the classification of C^*-algebras – Minimal dynamics

In my second lecture, I will focus on étale groupoids associated to minimal dynamical systems. I will discuss joint work with Robin Deeley and Ian Putnam where we construct various minimal systems and associated groupoids whose Minimal dynamics-algebras realize certain prescribed Elliott invariants. Along the way we show that given any finite CW complex, we can find a infinite compact metric space, with the same K-theory, which admits a minimal homeomorphism.

Enrique Pardo (Universidad de Cádiz)

Tight groupoids and their properties

We will explain how to construct the Exel's tight groupoid $\mathcal{G}_{\text{tight}}(S)$ associated to an inverse semigroup S. After that, we will explain how to characterize properties enjoyed by this groupoid, associated to characterization of algebraic properties of its groupoid algebra, in terms of intrinsic properties of the inverse semigroup.

This is a joint work with Ruy Exel (Universidade Federal de Santa Catarina, Brazil), published in:

> R. Exel, E. Pardo, The tight groupoid of an inverse semigroup, *Semigroup Forum* **92** (2016), 274–303.

Algebras associated to tight groupoids

We will present three examples of application of the techniques developed in the previous talk to study properties of three classes of algebras given in terms of generators and relations.

These are joint works with Pere Ara and Joan Bosa (Universitat Autònoma de Barcelona, Spain), Toke Meyer Carlsen (University of the Faroe Islands, Denmark), Lisa Orloff Clark (Victoria University of Wellington, New Zealand), Ruy Exel (Universidade Federal de Santa Catarina, Brazil), Eduard Ortega (NTNU Trondheim, Norway and Aidan Sims (University of Wollongong, Australia), published in:

> R. Exel, E. Pardo, Self-similar graphs, a unified treatment of Katsura and Nekrashevych C*-algebras, *Adv. Math* **306** (2017), 1046–1129.

> T.M. Carlsen, E. Ortega, E. Pardo, C*-algebras associated to Boolean dynamical systems, *J. Math. Anal. Appl.* **450** (2017), 727–768.

L.O. Clark, R. Exel, E. Pardo, A Generalised uniqueness theorem and the graded ideal structure of Steinberg algebras, *Forum Mathematicum* **30(3)** (2018), 533–552.

P. Ara, J. Bosa, E. Pardo, A. Sims, The groupoids of adaptable separated graphs and their type semigroups, arXiv:1904.05197v1 (2019).

Contributed talks

Becky Armstrong (University of Sydney)

Simple graph algebras

It is well known that the Cuntz–Krieger algebra associated to a row-finite, source-free directed graph is simple if and only if the graph is cofinal and every cycle in the graph has an entry. In this talk I will describe the results of my PhD research, in which I characterise simplicity of twisted C^*-algebras of topological higher-rank graphs in terms of the underlying graphical and cohomological data. These C^*-algebras are constructed using groupoid techniques for the purpose of this simplicity characterisation, but I will also briefly describe two product-system models for twisted C^*-algebras of topological higher-rank graphs. (This is joint work with my PhD supervisors, Nathan Brownlowe and Aidan Sims.)

Tristan Bice (Cardinal Stefan Wyszyński University)

Étale groupoid dualities

We discuss several general ways of using inverse semigroups to construct locally compact étale groupoids by utilising ideas from point-free topology. These constructions include non-ample groupoids which could thus provide combinatorial groupoid models even for projectionless C^*-algebras like the Jiang–Su and Jacelon–Razak algebras. Moreover, these constructions can be beefed up to dualities encompassing Lawson's non-commutative Stone duality as well as Exel's tight groupoid construction. We also outline similar constructions from more general $*$-semigroups, which could be applied to construct the Weyl groupoid just from the $*$-semigroup structure of the normalisers relative to a Cartan subalgebra, simplifying the original constructions of Kumjian and Renault as well as more recent generalisations by Exel and Pitts. This, in turn, has the potential to yield a fully functorial non-commutative extension of the classic Gelfand duality.

Chris Bourne (Tohoku University)

Groupoid cocycles and Delone dynamical systems

Given an étale groupoid and continuous $\mathbb{R}^d$-valued cocycle, we construct a bimodule over the unit space encoding the cocycle dynamics. A key motivating example are dynamical systems associated to Delone sets. We then consider some index theoretic applications of this bimodule. Time permitting, we will also consider Delone sets with finite local complexity, where more refined invariants can be defined. This is joint work with Bram Mesland.

Kristin Courtney (WWU Münster)

Twisted Steinberg algebras

Since their introduction in 1980 by Renault, twisted groupoid C^*-algebras have proven to be extremely valuable in studying the structural properties for large classes of C^*-algebras. Interest has grown even more in recent years since Barlak and Li's 2015 article which revealed

deep connections between these C^*- algebras and the UCT problem from the classification program for C^*-algebras.
Meanwhile, in the non-twisted setting, Steinberg algebras associated to Hausdorff ample groupoids have gained popularity as sources for interesting purely algebraic examples as well as insight into groupoid C^*-algebras. Thus, a twisted notion of Steinberg algebras provides a welcome bridge between algebraic and analytic results for twisted groupoids.
In this talk, I will give an introduction to twisted Steinberg algebras. This is joint work with Becky Armstrong, Lisa Orloff Clark, Ying-Fen Lin, Kathryn McCormick, and Jacqui Ramagge.

Jamie Gabe (University of Wollongong)

The extension problem for graph C^*-algebras

I will give a complete K-theoretical description of when an extension of two simple graph C^*-algebras is again a graph C^*-algebra.
This is joint work with Søren Eilers, Takeshi Katsura, Efren Ruiz, and Mark Tomforde.

Leonard Huang (University of Nevada)

Cocycles on Deaconu-Renault groupoids and KMS states for generalized gauge actions

Let k be a positive integer, X a compact metrizable space, and $\sigma = (\sigma_i)_{i\in[k]}$ a family of k commuting local homeomorphisms on X. One can construct an étale locally-compact Hausdorff groupoid, $\mathcal{G}(X,\sigma)$, whose space of objects is

$$\left\{(x,\boldsymbol{l},y) \in X \times \mathbb{Z}^k \times X \;\middle|\; \left(\exists \boldsymbol{m},\boldsymbol{n} \in \mathbb{N}_0^k\right)\left(\boldsymbol{l} = \boldsymbol{m} - \boldsymbol{n} \text{ and } \sigma^{\boldsymbol{m}}(x) = \sigma^{\boldsymbol{n}}(y)\right)\right\}.$$

This groupoid is known as the **Deaconu-Renault groupoid** of (X,σ). If c is a continuous $\mathbb{R}$-valued 1-cocycle on $\mathcal{G}(X,\sigma)$, then we obtain a family $\varphi = (\varphi_i)_{i\in[k]}$ of k elements of the Banach space $\mathrm{C}(X,\mathbb{R})$ of functions via

$$\forall i \in [k],\ \forall x \in X: \qquad \varphi_i(x) := c(x,\boldsymbol{e}_i,\sigma_i(x)).$$

It can be shown that the family φ necessarily satisfies a set of k algebraic relations — collectively called the **Cocycle Condition** — that involves the family σ.
In this talk, I shall present a new result that says: Given a family $\varphi = (\varphi_i)_{i\in[k]}$ of k elements of $\mathrm{C}(X,\mathbb{R})$, the Cocycle Condition is also sufficient for φ to give rise to a continuous $\mathbb{R}$-valued 1-cocycle on $\mathcal{G}(X,\sigma)$. This yields an algebraic classification of the set $\mathcal{Z}^1_{\mathrm{cont}}(\mathcal{G}(X,\sigma);\mathbb{R})$ of all continuous $\mathbb{R}$-valued 1-cocycles on $\mathcal{G}(X,\sigma)$. The result also says that the Cocycle Condition is equivalent to the commutativity of the family $\left(R^X_{\sigma_i,\varphi_i} : \mathrm{C}(X,\mathbb{R}) \to \mathrm{C}(X,\mathbb{R})\right)_{i\in[k]}$ of k Ruelle transfer operators, which offers a nice connection to ergodic theory.
Using this classification result and a well-known theorem of Sergey Neshveyev, it is possible to classify, for every $\beta \in \mathbb{R}$ and every $c \in \mathcal{Z}^1_{\mathrm{cont}}(\mathcal{G}(X,\sigma);\mathbb{R})$, the set of all KMS_β-states on $C^*(\mathcal{G}(X,\sigma))$ for the generalized gauge action corresponding to c — in terms of Ruelle transfer operators.
I shall also present a version of the Ruelle-Perron-Frobenius Theorem for a family of commuting Ruelle transfer operators. With this theorem, one can show, under certain mild conditions on σ and on φ, that a Ruelle-Perron-Frobenius probability eigenmeasure on X exists that allows us to construct KMS states on $C^*(\mathcal{G}(X,\sigma))$ for generalized gauge actions coming from φ.
Applications involving discrete and topological higher-rank graphs shall be shown. It is hoped that the ideas introduced in this talk will appeal to C^*-algebraists and ergodic theorists alike.
This is joint work with Carla Farsi, Alex Kumjian, and Judith Packer.

Yusra Naqvi (University of Sydney)

Reconstructing simplicial group actions

Many simplicial complexes which are important from a combinatorial, geometric or representation-theoretic perspective have enormous symmetry groups. Associated to the action of a group G on a simplicial complex X, one has the (typically much smaller and simpler) quotient space X/G. This quotient does not, by itself, carry enough structure to reconstruct either X or the G-action. The purpose of this talk is to describe what additional algebraic data is needed in order to perform such a reconstruction, using algorithms are derived from generalisations of the classical Bass-Serre theory for reconstructing group actions on trees. This talk is based on joint work with Lisa Carbone and Vidit Nanda.

David Roberts (University of Adelaide)

Groupoids, spans and cospans

Groupoids of many flavours (Lie, topological, locally compact) have the property that certain maps between them should be considered as equivalences, but which are not. To get around this one can localise the 2-category of your favourite groupoids at these maps. One way to do this is to consider "right fractions"—spans $X \leftarrow X' \to Y$—and another way is to consider "left fractions"—cospans $X \to Y' \leftarrow Y$—where in both cases the wrong-way leg is from the class to be inverted. This talk will show that this procedure can be performed with minimal assumptions on the class of groupoids. The right fractions case appears implicitly in the construction of the linking groupoid for a Morita equivalence. The case of left fractions generalises a special case of a construction of Land, Nikolaus and Szumio as applied to discrete groupoids, and the problem of functoriality of their C^*-algebras.

Dave Robertson (University of Newcastle)

Locally compact piecewise full groups

A group G acting faithfully by homeomorphisms of the Cantor set is called piecewise full if any homeomorphism assembled piecewise from elements of G is itself an element of G. I will discuss when such a group admits a non-discrete locally compact second countable group topology and describe a number of examples. This is joint work with Alejandra Garrido and Colin Reid.

Camila Fabre Sehnem

Amenability for Fell bundles extended from free semigroups

Fell bundles may be viewed as generalised partial group actions. Under mild assumptions, a Fell bundle comes from a partial group action, and if it is saturated, the action is global. Amenability for Fell bundles is defined so that it generalises amenability of groups. In this talk, we will introduce a notion of Fell bundles extended from positive cones of quasi-lattice orders. That is, they arise from certain actions of semigroups. This includes semi-saturated Fell bundles over $\mathbb{Z}$ and semi-saturated and orthogonal Fell bundles over free groups as considered by Exel. We will describe cross sectional C*-algebras of Fell bundles extended from free semigroups and from Baumslag–Solitar semigroups as Cuntz–Pimsner algebras. We then use gauge-equivariant uniqueness theorems for Cuntz–Pimsner algebras to prove amenability for such Fell bundles.

Adam Sierakowski (University of Wollongong)

Dimension theory for k-graph C^*-algebras.

I will present at least one reason why P-graphs are interesting. Using P-graphs one can characterise when a k-graph C^*-algebra has topological dimension zero in terms of the underlying graph. This is joint work with D. Pask and A. Sims.

Jack Spielberg (Arizona State University)

C^*-algebras of monoids related to Artin–Tits groups

If the relations of a group presentation involve only positive powers of the generators then an abstract monoid is also defined by the same presentation. In general one cannot expect that the natural homomorphism from the monoid to the group be injective. However there are conditions on the presentation that imply this injectivity, as well as conditions guaranteeing other properties of the monoid. We use these to study the C^*-algebras associated to certain submonoids of Artin–Tits groups. In some cases we discovered a directed graph whose C^*-algebra is a crucial element of our methods. More generally, an undirected labeled graph (the Coxeter diagram) can be used. This is joint work with Xin Li and Tron Omland.