Novel threshold concepts in the mathematical sciences

David Easdown, School of Mathematics and Statistics, University of Sydney
Leigh Wood, Faculty of Business and Economics, Macquarie University

Abstract: *The purpose of this research is to identify and examine some unusual, unexpected or novel threshold concepts in mathematics. This is ongoing work that contributes towards the development of a suite of resources and strategies for helping learners in the mathematical sciences work towards fulfilling their potential by successfully negotiating and passing through liminal spaces. This project is an offshoot of an ongoing Professional Development Unit for training tertiary mathematics teachers, initiated by the second author, supported by the (former) Australian Learning and Teaching Council, and sponsored by the Australian Mathematical Society. This Unit is linked to the Society's website.*

Introduction

Threshold concepts appear to be pivotal in formative processes that lead to positive or negative dispositions towards mathematics, especially from early childhood, though 'rebirthing' can occur at any age. On the positive side, one hears of so-called 'eureka moments', typified by the legendary Archimedes leaping out of his bath after making a monumental discovery about the displacement of volumes in a liquid. There is no doubt that Archimedes (almost certainly an apocryphal representation of a whole school of ancient Greek scholarly enquiry) passed through a portal, or sequence of portals opening up to ever-increasing vistas, and used such physical or geometrical experiments, combined with logical thinking, to develop or spawn an entire science. The mathematics of area and volume, and relationships with irregular patterns or shapes, has intrigued mathematicians for thousands of years, culminating, in the seventeenth century, in the extraordinary discovery of the close relationship between the differential calculus and integration. An irrevocable explosion of progress in the mathematical sciences followed.

On the negative side, however, it is common to hear of people who have been irreparably damaged by an experience, stopped in their tracks or slipped and fell over, losing confidence in their mathematical ability and developing a distaste or abhorrence towards even the simplest kinds of quantitative or algebraic reasoning. How tragic it is for such people never to experience, or get to the point of experiencing, the Archimedean feeling of elation or sense of transformation that is possible through mathematics. Courant and Robbins (1941), in the introduction to their classic text *What is mathematics?: an elementary approach to ideas and methods*, write that everyone should learn calculus, and that the associated mathematics is within the grasp of every educated and informed person, who desires knowledge and an appreciation of the greatest achievements of our civilization.

From the point of view of mathematics, William Rowan Hamilton (1805-1865) is surely a candidate for Ireland's Archimedes. In 1843, whilst walking along a Dublin canal with his wife, he discovered the celebrated equations describing the arithmetic of the quaternions "$i^2 = j^2 = k^2 = ijk = -1$". He felt so moved by the occasion that he inscribed them in stone on Broom Bridge, a few kilometers away from Trinity College, where he had been a student, and from the present day Hamilton Building, named in his honour (and where the 4th Biennial Threshold Concepts Conference was held, in June 2012).

These are remarkable equations. They establish the foundations of a well-defined arithmetic on a four-dimensional vector space that turns out to be non-commutative. The real numbers form a one-dimensional arithmetic, and the complex numbers a two-dimensional arithmetic. Both of these are examples of *fields*.

In the early nineteenth century, it was an important open problem to determine whether there exists a field in three dimensions, extending real arithmetic. Hamilton attempted for many years to create such an arithmetic, and came to a sudden realisation that the attempt was in vain: the next smallest dimension for which an arithmetic existed was four (not three), and commutativity was necessarily lost. He discovered *division rings* and founded non-commutative algebra, with far-reaching applications to mathematics and physics (and now also in present-day computer science and engineering). His tortured pathway, backwards and forwards through liminal space, is well documented through letters and correspondence with family, friends and other mathematicians and scientists (Graves (1975)). Glynis Cousin (2012) raises the question whether liminality is about space or relationships, and in a mathematical sense Hamilton tells us that it is both simultaneously: his discovery is a prototype for the sublime interplay between freedom and constraint, and the delicate balance between imagination, fantasy and mathematical reality that confronts all serious researchers in mathematics.

There is a paucity of research specifically on threshold concepts in mathematics, the remedy of which we hope might be stimulated by some of the examples in the following sections. The role of proof as a pivotal threshold concept in mathematics has been explored in Easdown (2007), from both heuristic and more formal technical perspectives, and Jooganah (2009), with sociological implications. The notion of a function, and the interplay between *functions as processes* and *functions as objects*, has been explored by Pettersson (2011), who conducts a detailed longitudinal study involving a prospective mathematics teacher. This has the advantage of viewing progress backwards and forwards through liminal space, from both the perspective of a learner in the discipline of mathematics and someone aspiring to become a mathematics educator. Indeed, this dichotomy, between learner and educator, is a constant theme in the training of primary teachers by Jacqui Ramagge (2010) and her team, and for tertiary mathematics educators, by Wood *et al.* (2010) (and see also Brown *et al* (2010)), through workshops and online learning modules, stemming from the (former) Australian Learning and Teaching Council Project *Effective Teaching, Effective Learning in the Quantitative Disciplines* (now accessible online through the website of the Australian Mathematical Society).

We offer a sprinkling of contrasting and seeding examples that fall, roughly speaking, into one of the following categories: *formative* stage (primary or early childhood); *middle developmental* stage (secondary or early tertiary); *professional or meta-cognitive* stage (late tertiary to postgraduate). In each case, the examples exhibit characteristics of being *transformative*, *troublesome* or *counter-intuitive*, *integrative* and *irreversible.* They also involve some kind of journey that begins with a pre-liminal state of aimlessness or incomprehension. The person may remain trapped there, possibly rebounding or repelled from liminal space. Alternatively, the person may successfully pass through liminal states of suspension or instability, culminating in a post-liminal state of empowerment or ‘enlightenment’. The final extended illustration below is one of the most striking examples of tortured, convoluted movement through pre-liminal and liminal space, by eminent mathematicians and philosophers. It culminated in a sudden explosion or revolution of ideas and progress in modern mathematics. This serves as a tribute, on the centenary of his birth, to Alan Turing (born 23 June 1912), one of the most creative and original thinkers of the first half of the twentieth century.

Primary or early childhood or stages

Ross Fitzgerald (2010), in his autobiography, describes a personal anecdote of the role of zero in putting him off mathematics at a young age. He came to school with oranges in his bag, and showed his teacher. He put out his right hand holding all three, and also held out his left hand with none. He then asked a simple question: “If I multiply the no oranges by the three oranges, where do they go?” The teacher could not explain where the oranges went and destroyed a small boy’s confidence in mathematics. Ross describes this as a pivotal moment that turned him away from mathematics permanently. (He also notes that he discovered much later that the primary school teacher had not been trained in mathematics.) In the language of threshold concepts, Ross had arrived at school in a pre-liminal state, possibly with high hopes and even enthusiasm. His interaction with the teacher entered a slippery liminal state that unfortunately led to repulsion and a disastrous outcome. What was the underlying threshold in this example? Clearly the young boy understood the notion of zero, or “nothingness”, but the issue was how zero interacted with *multiplication of numbers.* Multiplication is a highly nontrivial operation in general and exceedingly difficult to explain to a young child, and takes careful thought and preparation even for integers.

Ramagge (2010) emphasises the importance of training primary school teachers in mathematical thinking, not so much for the knowledge content, but to consolidate their own understanding. Even when teachers pass through portals, there needs to be self-reflection about the journey, in order to successfully guide others. Stephen Brookfield (1995) describes an incident when he wanted to learn to swim from an expert, who, despite his obvious prowess, was incapable of explaining basic steps, or even seeing the problem from the point of view of a complete novice. This is related to the *Principle of Reflected Blindedness* (introduced by Easdown (2006)), interpreted in terms of the interplay between syntax and semantics, which says our own profound or ingrained knowledge or expertise can make us blind to the point of view of the learner.

The behaviour of numbers and the way they interact leads to abstract algebra, and Easdown (2011a) recalls a pivotal moment for him that led to a life-long love of mathematics and cemented at a very early age the notion that mathematics is all-powerful and provides keys to unravelling mysteries. The teacher had entered the room of third-graders and gave a sequence of instructions: "Think of a number between one and ten. Keep it secret. Double it. Add four. Halve what you now have. Subtract the secret number you started with. You are now thinking of the number two!" This was astonishing to a small child and the teacher used it to illustrate a key concept in algebra. If the secret number is x then the teacher's instructions were to progressively evaluate "$(2x+4)/2-x$", which always simplifies to the number 2, regardless of the choice of secret number. Several pathways through liminal space opened up: first, the teacher was consolidating simple arithmetic, not as a chore or boring exercise, but as a means to an end (to read minds); second, he surprised all of the class and piqued their interest, savouring the sensation of suspension and uncertainty; third, and most importantly, he explained how one could represent a number as a symbol, and by manipulating symbols produce an astonishing conclusion. This last aspect creates a hybrid of threshold concepts: an unknown value x, algebraic manipulation and proof. Separately, each ingredient may seem unremarkable or uninteresting, but brought together they create a dynamic that takes the learner towards and through a portal that opens up to a fascinating landscape. After this demonstration by the teacher, the third-graders were switched on and excited. They subsequently created their own more elaborate mind-reading games, actively seeking out problems for which "solve for x" or "get rid of x" became tantamount.

Secondary to early tertiary stages

The ability to add, subtract, multiply and divide, and understand when each operation is appropriate and effective, is, of course, taught from elementary primary school. However, Frank Quinn (2011) writes in detail about the vagaries of teaching fractions (directly related to division), which he describes as a "perennial source of trouble". His examples demonstrate clearly that the learner must negotiate many twists and turns before reaching a portal that can come in many different guises. The problems this creates in training teachers is explored further by Wu (2011), who explains the gulf between effective abstract mathematics and what is possible to convey in the classroom. Algebraic manipulation is one of the core devices for producing mathematics, and solving equations in particular, and there comes a point, somewhere between secondary and even tertiary education, where a student has to develop awareness of the processes involved and not simply react intuitively and "hope for the best".

Easdown (2011b) was initially surprised when a group of university students asked him how to get beyond an apparently plausible manipulation they had made in attempting to complete an assignment:

> "We are trying to solve for t given the equation $2P = P(1 + i/100)^t$. We take P away from both sides to get $P = (1 + i/100)^t$, and now are stuck! Help!"

The students were attempting to isolate t with a view to taking logarithms and rearranging, to explain the well-known *Rule of Seventy* used by investors to estimate the time it takes to double an investment compounded at i per cent annually (taking approximately $70/i$ years). Unwittingly they had used subtraction of P on the left-hand side instead of division by P (used correctly on the right-hand side). This simple error made the problem essentially meaningless and impregnable. It was as though a door had slammed shut on a distant portal, and no light remained. Fortunately for them, a nudge from the lecturer, pointing out that they had confused and compounded subtraction and division in the same manipulation, unblocked the pathway and they were able to return to liminal space and successfully solve the problem. Success or failure in formal mathematics is sensitive to even the slightest perturbations. Debugging computer programs, for example, is notoriously difficult, and mathematics has a similar sensitivity, certainly at the level of syntax of formulae. Even if an error is trivial, finding it can be like searching for a needle in a haystack, and exacerbated if, through inexperience, one doesn't know in advance what the needle looks like!

Walter Bloom, in one of the workshops (Easdown (2011a)), gives a beautiful example of the ambiguity of arithmetic operations:

$$\text{(One haystack)} + \text{(One haystack)} = \text{(One haystack)}$$

A young person can understand the heuristic conveyed by this equation: if one pools two haystacks together one gets just another single haystack (albeit larger). If size is not being modelled by the mathematics, then this makes perfect sense, and it leads to a variation of the operation of addition. Indeed with usual addition of integers we have 0+0=0, but the haystack example suggests even the possibility of, say 1+1=1, and that leads to the notion of *semiring arithmetic*, important in theoretical computer science and applications to algebraic geometry, in particular the emerging field of *tropical geometry*. Beyond this threshold, the mathematical scenery becomes vast and exotic, and examples such as this one of Bloom's provide simple entry points for a curious student.

Professional or Meta-cognitive stages

The hallmark of a successful professional mathematician is the ability to step back and think about mathematics as a whole and seek relationships. Stated this way, this may seem trite and obvious, but in fact many students get embroiled in detail and find themselves stuck or bogged down in the unistructural and multistructural phases of the SOLO taxonomy (see Biggs and Collis (1982) and Biggs and Tang (2007)), without passing into the relational or extended abstract phases. The notion of *movement through the boundaries* between phases is closely related to the successful passage through liminal spaces, and becomes a threshold concept itself. Easdown (2007) explains the way this malleability of boundaries is exploited by practising mathematicians, by free and unashamed use of the so-called *Plateau Principle: look for and be prepared to use a variety of plateaus as starting points for a mathematical investigation.* He likens this to using a helicopter to fly to the top of a glacier, in order to embark on an amazing skiing journey, rather than first trudging through the ice flows with the constant risk of slipping into a crevasse. Flying over terrain in a helicopter is not cheating, but a recognition of the power of utilising what others have invented or achieved in the past. Isaac Newton famously stated "If I have seen further than others, it is because I have stood on the shoulders of others." The entire feat of strategically negotiating liminal space and finding portals that one knows in advance exist is one of the most effective of threshold concepts in mathematics and involves meta-cognition: the ability to reflect about what one is doing in relationship to what others are doing or have done.

Meta-cognitive threshold concepts are especially important in higher mathematics and we give an extended example that relates to the foundations of mathematics. Gottlob Frege (1903), a philosopher and mathematician, set himself the task of developing what he believed to be unequivocal and apparent "self-evident" premises on which to base set theory. His so-called *Comprehension Principle* states that any set may be defined by a well-formulated condition, using whatever symbolism is appropriate in the context. This principle is used so widely and commonly in mathematics that it seems unassailable and impregnable. However Bertrand Russell, in 1901 (see Von Heijenoort (1967)), had sent Frege a note:

Consider the set S of sets that are not members of themselves. Is S a member of itself?

This is now famously referred to as *Russell's paradox*: the set *S* cannot be a member of itself, as this leads to a contradiction, so it therefore must be a member of itself, which also leads to a contradiction. There is no way out of this dilemma unless one abandons the notion that *S* is a set. Frege (1903), in his major treatise *Grundgesetze der Arithmetik,* then about to go to press, realised his Comprehension Principle was on quicksand, as it was capable of allowing fallacies, and published an addendum:

> *Hardly anything more unwelcome can befall a scientific writer than that one of the foundations of his edifice be shaken after the work is finished. I have been placed in this position by a letter of Mr Bertrand Russell just as the printing of this volume was nearing completion.*

Russell felt so bad about this turn of events, caused by the discovery of his paradox, that he and Alfred Whitehead spent the next decade developing their theory of types, intended as a solid foundation of set theory (and all of mathematics) that avoids paradoxes, in their major work *Principia Mathematica* (Whitehead and Russell (1913)). Frege had firmly believed himself to be taking the academic world through a major portal with the development of the foundations of mathematics, only to have the door slammed in his face, then sliding backwards through liminal space to a chaos from which he never fully recovered. Russell and Whitehead however struggled for many years through their own liminal spaces and published their own treatise in an incomplete form (*Principia* was intended to be four volumes, but only three were finished). Nevertheless they created their own "shoulders of giants" and Kurt Gödel (1931) was to pick up on their main ideas and forge his celebrated and revolutionary Incompleteness Theorem, which tells us that number theory is essentially incomplete, in the sense that true statements can be formulated in number theory that cannot be proved within number theory (but only by stepping outside number theory). The proof relies on setting up within number theory the so-called *Gödel statement*, which is essentially an elaborate reformulation of Russell's Paradox:

This statement is unprovable.

This self-reflective statement (accepting that it can be formulated somehow in number theory) must be true, by the soundness of first order logic, and therefore, at the same time, unprovable. Gödel (1929) had earlier, in his doctoral thesis, proved the completeness of first order logic (all statements that are true in all models are provable in first order logic) and it was an open problem whether provability was decidable, that is, whether a simple algorithm exists for deciding whether any give statement in first order logic is provable or not. This is the famous *Entscheidungsproblem* of David Hilbert (see Hilbert and Ackermann (1928)). For example, monadic first order logic (that uses only predicates, not binary or other multivalued relations) had been proven to be decidable by Lowenheim (1915) and there appeared to be major stumbling blocks to taking this further. The first thirty years of the twentieth century were a seething whirlpool of activity amidst uncertainty, as people strove towards false hopes and conclusions, with only partial results and shifting sands, with elusive apparitions of portals in the distance. In 1936 Alonzo Church (1936) published a negative answer to Hilbert's question, that indeed first order logic is undecidable. His method used the so-called *lambda calculus*, which was equivalent to another invention by one of Church's students, Alan Turing, the so-called *Turing machines.* Turing had also proved, independently of Church, the negative answer to the Entscheidungsproblem, and rushed his answer into print (Turing (1937)). He first set up the so-called *Halting Problem* and proved this to be unsolvable: no algorithm exists that can decide whether any given Turing machine halts with any given input. Turing then showed that the logical consequences of the list of instructions defining any Turing machine may be encoded using the language of first order logic. This implies that any decision algorithm for first order logic necessarily entails a decision algorithm for the Halting Problem, which does not exist. Hence no decision algorithm for provability of first order logic exists. This method yields a template for undecidability results in general that has become the golden standard ever since:

> *To prove that a given problem in mathematics is undecidable, reduce it to the Halting Problem for Turing machines.*

The unsolvability of the Halting Problem is one of the grandest portals in modern mathematics. The key that unlocks the door is the idea of self-reference, in this case a machine that looks at its own instructions, the ultimate navel-gazer. Turing's lock and key were forged in the sea of liminality that evolved from the extraordinary saga of Russell and Frege (superbly paraphrased and illustrated in *Logicomix* by Doxiadis and Papadimitriou (2009)).

Conclusion

It has not been the purpose of this paper to preach about mathematics: good mathematics speaks for itself. However, we have offered examples and anecdotes that are novel in nature, with the intention of seeding or provoking reactions from the reader or from participants in our workshops and training sessions for tertiary mathematics instructors and educators. John Dewey (1933) advises that we do not learn from experience but rather from *reflecting* on experience. Brian Foley (2012) refers to this process as *meta-learning* and applies the idea systematically and successfully to pedagogy in engineering. Land *et al* (2005) emphasise the naturally oscillatory or recursive behaviour that typifies successful movement between pre-liminal and liminal phases. Recursion is, in essence, reflective or self-referential. Cousin (2006) distinguishes this dynamic and often troublesome or unsettling process from the more counterfeit alternative of superficial 'mimicry' (which is, unfortunately, often encouraged and even rewarded). Plato tells a pupil to "know thyself" (Jowett (1892)), with the intention of developing the habit of lifelong, self-reflective learning. In the spirit of Plato, we have invited and continue to invite people, from all backgrounds and interests, to engage in profitable introspection of their mathematical experiences and

1. *Offer their own examples where they have been switched on or turned off mathematics by some pivotal incident that behaved like a threshold or impenetrable barrier.*
2. *Explore underlying reasons for the importance of such incidents and how they relate to threshold concepts, especially the means by which learners become stuck or repelled, but nevertheless find their way through liminal spaces.*
3. *Make suggestions about how these incidents and experiences can inform practice towards improving the teaching and learning of mathematics*. (Easdown (2011a))

This models our own practice in university classrooms where we encourage tertiary students to confront their prior learning in mathematics.

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